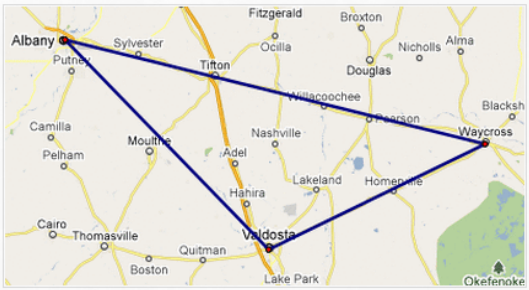
5-2 Bisectors of Triangles



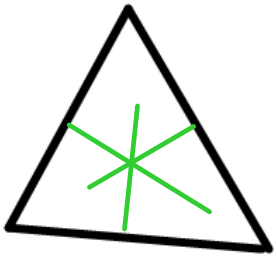
Today you will learn how to place a business so that it is the same distance to three main roads and how to place a distribution center equidistant to 3 main factories in 3 different city's like shown on the map to the left

Objectives

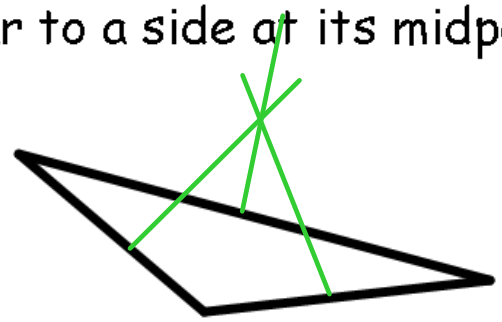
Prove and apply properties of perpendicular bisectors of a triangle.

Prove and apply properties of angle bisectors of a triangle.

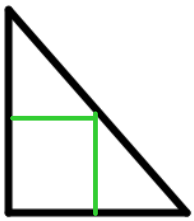
Perpendicular bisector of a triangle:
is perpendicular to a side at its midpoint.



Acute Triangle



Obtuse Triangle



Right Triangle

Since a triangle has three sides, it has three perpendicular bisectors.

They don't always meet inside the triangle.

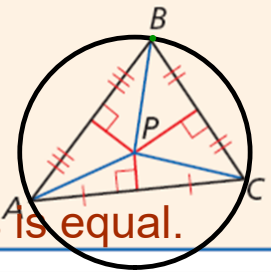
When you construct the perpendicular bisectors, you find that they have an interesting property.

Perpendicular bisectors meet at the **circumcenter of the triangle**.

Theorem 5-2-1 **Circumcenter Theorem**

The circumcenter of a triangle is equidistant from the vertices of the triangle.

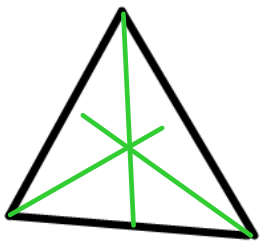
$PA = PB = PC$



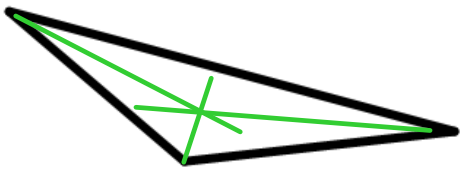
From the circumcenter (P) to the corners is equal.

Angle bisector of a triangle:

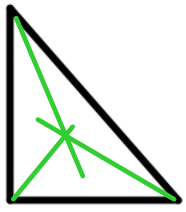
bisects the angles of a triangle.



Acute Triangle



Obtuse Triangle



Right Triangle

Unlike the circumcenter, the incenter is always inside the triangle.

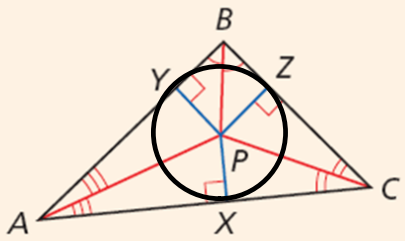
Angle bisectors meet at the **incenter of the triangle** .

Theorem 5-2-2 **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

$PX = PY = PZ$

From the incenter (P) to the sides is equal.



Remember!

The distance between a point and a line is the length of the perpendicular segment from the point to the line.

Example 1

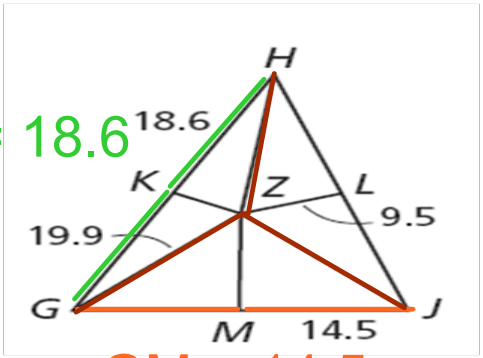
Use the diagram. Find *GM, KG and JZ.*

$\overline{KZ}$, $\overline{ZL}$ and $\overline{MZ}$ are perpendicular bisectors of ΔGHJ.

Since we are given perpendicular bisectors, Z is the circumcenter. The circumcenter (Z) is equal to the corners.

So JZ = 19.9

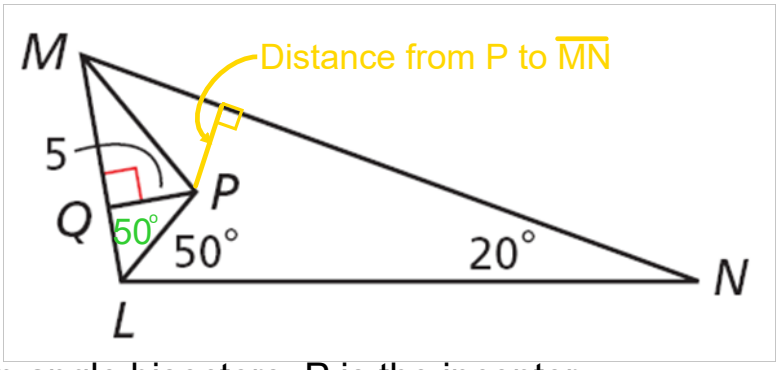
KG = 18.6



GM = 14.5

Example 3

$\overline{MP}$ and $\overline{LP}$ are angle bisectors of ΔLMN. Find the distance from P to $\overline{MN}$ and the $m\angle PMN$.

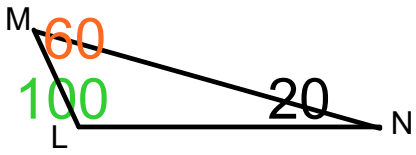


Since we are given angle bisectors, P is the incenter.
The incenter (P) is equal distance to the sides.

the distance from P to $\overline{MN}$ is 5.

To find angle's remember they add to 180

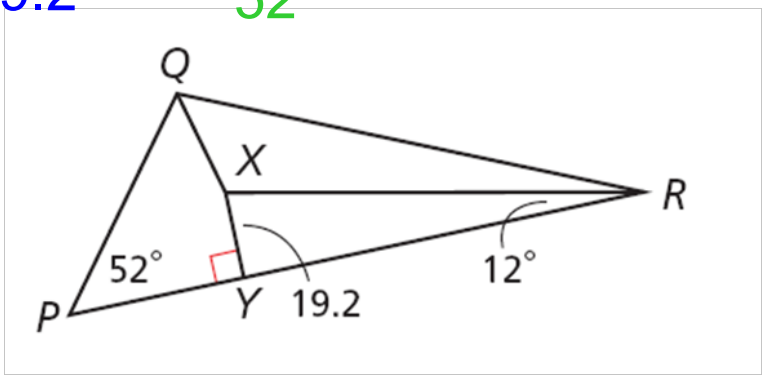
$m\angle PMN = 30^\circ$



Example 4

$\overline{QX}$ and $\overline{RX}$ are angle bisectors of ΔPQR. Find the distance from X to $\overline{PQ}$ and $m\angle PQX$..

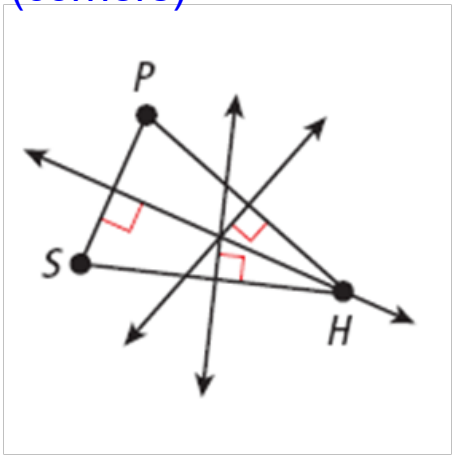
19.2 52°



A city planner wants to build a new library between a school, a post office, and a hospital. Draw a sketch to show where the library should be placed so it is the same distance from all three buildings.

(corners)

Draw the triangle formed by the three buildings. To find the circumcenter, find the perpendicular bisectors of each side. The position for the library is the circumcenter.



A city plans to build a firefighters' monument in the park between three streets. Draw a sketch to show where the city should place the monument so that it is the same distance from all three streets. Justify your sketch.

(sides)

The incenter of a Δ is equidistant from the sides of the Δ. Draw the Δ formed by the streets and draw the ∠ bisectors to find the incenter, point *M*. The city should place the monument at point *M*.